Single Trial Estimation of Evoked Potentials using Gaussian Mixture Models with Integrated Noise Component

Arthur Flexer[1], Herbert Bauer[2], Claus Lamm[2], Georg Dorffner[1]

[1] The Austrian Research Institute for Artificial Intelligence
Schottengasse 3, A-1010 Vienna, Austria
arthur@ai.univie.ac.at
[2] Department of Psychology, University of Vienna
Liebiggasse 5, A-1010 Vienna, Austria

Abstract. Gaussian Mixture Models with integrated noise component are a method developed for speech analysis to estimate signals hidden in background noise. We apply this technique to estimate single trial evoked potentials which are buried in noise up to five times stronger than the signal. An empirical study using artificial data is presented and results are shown to compare favourably to other techniques for single trial estimation.

1 Introduction

An evoked potential (EP) is the electro cortical potential measurable in the human electro encephalogram (EEG) before, during and after sensoric, motoric or cognitive events. An EP is defined as the combination of the brain electric activity that occurs in association with the eliciting event and 'noise', which is brain activity not related to the event together with inference from non-neural sources. Since the noise contained in single trial EPs is up to five times stronger than the signal, the common approach is to compute an average across several EPs recorded under equal conditions to improve the signal-to-noise ratio (SNR). For averaging of a set of N EPs, the assumed model for the ith EP is the following (see e.g. [Glaser & Ruchkin 1976]):

$$\vec{z}_i(t) = \vec{x}(t) + \vec{y}_i(t); \quad i = 1, 2, \dots N; \quad 0 \leq t < T \tag{1}$$

where $\vec{z}_i(t)$ is the ith recorded EP, $\vec{x}(t)$ the underlying signal, $\vec{y}_i(t)$ the noise associated with the ith EP, and T the duration over which each EP is recorded. The average $\hat{\vec{x}}(t)$ over the sample of N EPs is used to estimate the underlying signal $\vec{x}(t)$:

$$\hat{\vec{x}}(t) = \frac{1}{N}\sum_{i=1}^{N} \vec{z}_i(t) = \vec{x}(t) + \frac{1}{N}\sum_{i=1}^{N} \vec{y}_i(t) \tag{2}$$

Averaging will attenuate the noise $\vec{y}_i(t)$ and not the signal $\vec{x}(t)$ given that signal and noise linearly sum together to produce the recorded EPs $\vec{z}_i(t)$ and the evoked signal $\vec{x}(t)$ is the same for each recorded EP $\vec{z}_i(t)$ and the noise contributions $\vec{y}_i(t)$ can be considered to constitute statistically independent samples of a random process. If the above assumptions do not hold, the averaging will result in a biased, distorted estimate of the signal. This distorted estimate also prohibits analysis of unique trial to trial information, like different latencies or amplitudes of components in single trials. What is needed is a method which is able to estimate the underlying signal in single trial EPs.

The problem is closely related to the estimation of speech model parameters in noisy acoustic environments. [Rose et al. 1994] present a general framework of estimating integrated models of signal and background noise. They distinguish between 3 processes:

- **Z**: a process containing both the signal and the noise background, this is what usually can be directly observed (in our case this would be the single trial EP recordings).
- **X**: the signal alone, in speech analysis this could be recordings with very low or zero background activity.
- **Y**: the noise background alone, in speech analysis this could be segments of the recordings without any word utterances (at the beginning or in between).

The authors develop a general model around the assumption that all three processes can be modelled via mixtures of Gaussians. If **Z** can be observed it is necessary to either observe **X** or **Y** to be able to estimate the respective other "single" process. The authors present solutions for the case of additive Gaussian mixture models for **X** and **Y**. In this work we adopt the [Rose et al. 1994] approach to the problem of single trial estimation of evoked potentials.

Our work is structured as follows: first we describe a testbed of artificial EPs which allows for systematic variation of noise characteristics; then we present all technical details of the Gaussian Mixture Model with integrated noise component; in the results section we give quantitative measures for the achieved improvement in SNR; finally we discuss our approach by comparing it to other methods dealing with single trial estimation.

2 Data

The goal of our work is to devise a method that estimates EP signals hidden in considerable amounts of background noise. Since for real EP data the hidden signal is actually not known, we resort to simulated artificial EP data. Only the use of artificial data with known signal and noise proportions allows us to really quantify the gain in signal-to-noise ratio.

We simulated $500msec$ of a visually evoked EP recorded via $d = 22$ electrodes mounted according to the international 10-20 system. One artificial EP is of length $T = 125$ since we simulated a $250Hz$ recording. First we defined eight topographical patterns (each consisting of $d = 22$ values) corresponding to well

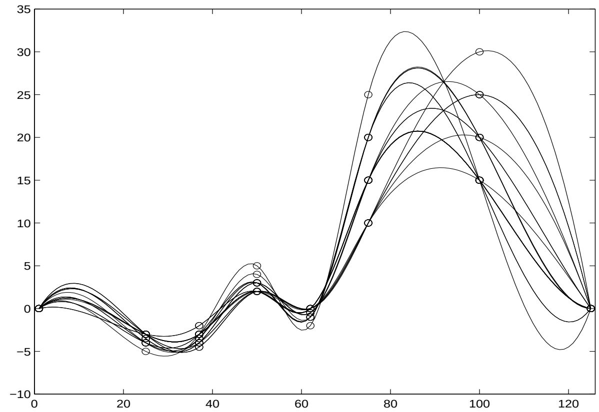


Fig. 1. All $d = 22$ channels of an artificial EP plotted in one graph (x-axis time, y-axis amplitude). The underlying topographical patterns are shown as circles at $t = 1, 25, 37, 50, 62, 75, 100, 125$. Note the earlier onset of the $P300$ at more frontal electrodes relative to more posterior electrodes (i.e. amplitude peaks of $\sim 30\mu V$ at $t \approx 80$ and 100).

known EP components like the $N100$ or $P300$. These topographical patterns have distinct amplitudesignal values and temporal latencies (see Fig. 1). By computing a cubic spline interpolation for each signal dimension d we obtain values at all 125 sample points. To simulate between-trial variation we randomly shifted the topographical patterns both in time and amplitude prior to spline interpolation (change of temporal position: $t_{new} = t_{old} + 3 * \mathcal{N}(0,1)$, change of amplitude: $x_{new} = x_{old} + 0.33 * \mathcal{N}(0,1) * \mid x_{old} \mid$, $\mathcal{N}$ is the Gaussian distribution).

We computed 5×100 of these artificial EPs with latency and amplitude variation. We added random Gaussian noise to each of these 5 EP sets in five different signal-to-noise ratios (SNR): 1:1, 1:2, 1:3, 1:4, 1:5. Gaussian noise was produced by limiting a $4000Hz$ Gaussian random signal to a bandwidth between 0 and $30Hz$ via FIR filtering and then down-sampling it to $250Hz$. A specific SNR was achieved by scaling the mean powers of signal and noise to yield the desired SNR. This yielded empirical mean *Input* SNR ratios of $3.34, -10.36, -18.45, -24.18$ and $-28.86dB$.

3 Methods

In the case of a simple Gaussian Mixture Model (GMM), the signal $\vec{x}$ is modelled as a mixture of Gaussians, independent in time:

$$p(\vec{x}|\lambda_s) = \sum_{i=1}^{M} p_i b_i(\vec{x}) \tag{3}$$

where the signal model $\lambda_s = \{p_i, \mu_i[d], \sigma_i[d]\}$ is a mixture of M Gaussian densities b_i with diagonal covariance matrix. The p_i are the mixing coefficients with $\sum_{i=1}^{M} p_i = 1$.

In the case of an GMM with integrated noise component by [Rose et al. 1994], the background noise $\vec{y}$ is also modelled as a mixture of Gaussians, independent in time:

$$p(\vec{y}|\lambda_b) = \sum_{j=1}^{N} q_j a_j(\vec{y}) \tag{4}$$

where the background noise model $\lambda_b = \{q_j, \mu_j[d], \sigma_j[d]\}$ is a mixture of N Gaussian densities a_j with diagonal covariance matrix. The q_j are the mixing coefficients with $\sum_{j=1}^{N} q_j = 1$.

The observable noisy signal $\vec{z}$ is given via a general function of signal and background noise:

$$\vec{z} = f(\vec{x}, \vec{y}) \tag{5}$$

The density of the noisy signal for state i and j at time step t is given by:

$$p(\vec{z}_t|i_t, j_t, \lambda_s, \lambda_b) = \iint_{C_t} b_{it}(\vec{x}_t) a_{j_t}(\vec{y}_t) d\vec{x}_t d\vec{y}_t \tag{6}$$

where C_t denotes the contour defined by Equ. 5. The likelihood of a noise corrupted observation $\vec{z}_t$ is given by:

$$p(\vec{z}_t|\lambda_s, \lambda_b) = \sum_{i=1}^{M} \sum_{j=1}^{N} p_i q_j p(\vec{z}_t|i, j, \lambda_s, \lambda_b) \tag{7}$$

In the following, reference to λ_b will be dropped as a notational shorthand. Therefore the signal model is simply $\lambda = \{p_i, \vec{\mu}_i, \vec{\sigma}_i\}$. Our goal is now to obtain estimates of the underlying signal model by maximizing the likelihood of a set of noisy observations Z. Estimation in the maximum likelihood sense involves finding the model λ which maximizes $p(Z|\lambda)$ with

$$P(Z|\lambda) = \sum_{I} \sum_{J} \iint_{C} P(X, Y, I, J|\lambda) dX dY \tag{8}$$

and

$$P(X, Y, I, J|\lambda) = \prod_{t=1}^{T} b_{it}(\vec{x}_t) p_{it} a_{j_t}(\vec{y}_t) q_{it}. \tag{9}$$

Application of the Expectation-Maximization (EM) technique [Dempster et al. 1977] yields the following general expressions for the estimation of the model parameters. Since all densities are assumed to have diagonal covariance matrices, each vector component can be estimated independently. The notational shorthands $\bar{p}_i$, $\bar{\mu}_i$ and $\bar{\sigma}_i$ therefore refer to arbitrary vector components.

$$\bar{p}_i = \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{N} p(i_t = i, j_t = j | z_t, \lambda) \tag{10}$$

$$\bar{\mu}_i = \frac{\sum_{t=1}^{T} \sum_{j=1}^{N} p(i_t = i, j_t = j | z_t, \lambda) E\{x_t | z_t, i_t = i, j_t = j, \lambda\}}{\sum_{t=1}^{T} \sum_{j=1}^{N} p(i_t = i, j_t = j | z_t, \lambda)} \tag{11}$$

$$\bar{\sigma}_i^2 = \frac{\sum_{t=1}^{T} \sum_{j=1}^{N} p(i_t = i, j_t = j | z_t, \lambda) E\{x_t^2 | z_t, i_t = i, j_t = j, \lambda\}}{\sum_{t=1}^{T} \sum_{j=1}^{N} p(i_t = i, j_t = j | z_t, \lambda)} - \bar{\mu}_i^2 \tag{12}$$

where $p(i_t = i, j_t = j | z_t, \lambda)$ is given by

$$p(i_t = i, j_t = j | z_t, \lambda) = \frac{p(z_t | i_t = i, j_t = j, \lambda) p_i q_j}{\sum_{i=1}^{M} \sum_{j=1}^{N} p(z_t | i_t = i, j_t = j, \lambda) p_i q_j}. \tag{13}$$

If we assume an additive noise model with $f(x_t, y_t) = x_t + y_t$ and a background process which can be modelled by a single Gaussian (i.e. $N = 1$ in all above formulas), we get:

$$E\{x_t | z_t, i_t = i, \lambda\} = \frac{\sigma_i^2}{\sigma_i^2 + \sigma_b^2} \left[z_t + \left(\frac{\sigma_b^2}{\sigma_i^2} \mu_i - \mu_b \right) \right] \tag{14}$$

$$E\{x_t^2 | z_t, i_t = i, \lambda\} = \frac{\sigma_i^2 \sigma_b^2}{\sigma_i^2 + \sigma_b^2} + E\{x_t | z_t, i_t = i, \lambda\}^2 \tag{15}$$

$$p(z_t | i_t = i, \lambda) = \mathcal{N}[z_t, \mu_i + \mu_b, \sigma_i^2 + \sigma_b^2] \tag{16}$$

where $\mathcal{N}[z_t, \mu_i + \mu_b, \sigma_i^2 + \sigma_b^2]$ is the Gaussian distribution corresponding to the observed noisy signal. Since both the signal x_t and the background y_t are random variables with Gaussian distribution their sum is also a Gaussian distribution with the above parameters.

4 Results

We applied Gaussian Mixture Models with integrated noise component (GMM-noise) to all 5 data sets described in Sec. 2. We used mixtures of $M = 10$ Gaussians for the signal $\vec{x}$ and a single Gaussian ($N = 1$) for the background noise $\vec{y}$. Using mixtures of $M = 20$ Gaussians yielded only minor improvements. As an estimate of the noise mean μ_b and variance σ_b^2 we used the respective parameters of the residual noise obtained via averaging [1]. For each set of 100 artificial EPs we computed an average according to Equ. 2 and subtracted it from all 100 artificial EPs. The remainder is the residual noise.

Once a GMMnoise model is trained we can use the component densities given in Equ. 16 to weigh the means of the Gaussians $\vec{\mu}_i$ at each time step t and obtain a denoised estimate $\hat{\vec{x}}'_t$ of the signal:

$$\hat{\vec{x}}'_t = \sum_{i=1}^{M} p(\vec{z}_t | i_t = i, \lambda) \vec{\mu}_i \tag{17}$$

Since our data are artificial EPs we know exactly which signals $\vec{x}$ are hidden in the noisy signals $\vec{z}$. We can compare the denoised estimates obtained via GMMnoise as well as the common average given in Equ. 2 to these signals $\vec{x}$. We obtain noise residuals by subtracting the real signals $\vec{x}$ from the denoised signal estimates $\hat{\vec{x}}'_t$ and $\hat{\vec{x}}$. The ratio in dB between the real signals $\vec{x}$ and these noise residuals are called *Output* SNRs and are given in Tab. 1 for both GMMnoise and averaging. The same information is depicted in Fig. 2(a).

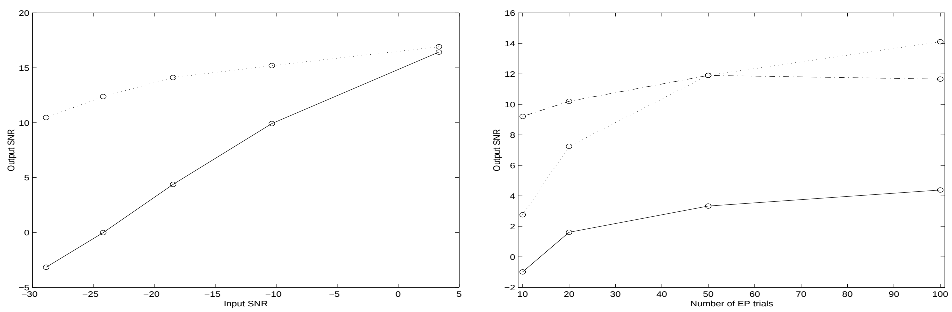
data set	1:1	1:2	1:3	1:4	1:5
Input SNR	3.34	– 10.36	– 18.45	– 24.18	– 28.86
Output SNR GMMnoise	16.43	9.92	4.38	– 0.02	– 3.17
Output SNR average	16.92	15.20	14.11	12.38	10.47

Table 1. Results for GMMnoise and averaging applied to the five EP data sets given as SNR in dB.

If the power of the underlying signal is approximately equal to the power of the background noise (data set "1:1"), GMMnoise is able to recover the information based on single EP trials as good as averaging which uses the full 100 trials of the data set. At lower SNRs, the performance of GMMnoise decreases relative to averaging. But even for data set "1:5", the improvement in SNR is around $25dB$. Our results are comparable to those achieved with other

[1] Since averaging does not fully remove the noise from the signal the variance of the noise residual is always underestimating the true noise variance. One could try to think about accounting for this to get more accurate estimates.

more involved approaches. [Cerutti et al. 1988] employ an ARX (autoregressive with exogenous input) model for single trial estimation using separate EEG and EOG (electro-oculogram) noise sources. The authors report an improvement in SNR of around $20dB$ for an Input SNR of $-30dB$ which is very similar to what GMMnoise achieves for data set "1:5". [Westerkamp & Aunon 1987] employ an a posteriori Wiener filter which is time-varying and uses information from multiple electrodes to estimate single trial EPs. Contrary to our GMMnoise this is a non-stationary technique taking into account temporal variations of noise and signal statistics. In their study on simulated EP data, the best improvement in SNR achieved for their lowest Input SNR of $-9dB$ is around $12dB$. GMMnoise achieved an improvement of almost $20dB$ for an Input SNR of $-10.36dB$ (see Tab. 1, data set "1:2").



(a) (b)

Fig. 2. (a) Input SNR (x-axis) vs. Output SNR (y-axis) in dB for GMMnoise (solid) and averaging (dotted). (b) Number of EP trials (x-axis) vs. Output SNR (y-axis) for GMMnoise (solid), averaging (dotted) and GMMnoise+ave (dash-dot).

Another application of GMMnoise besides estimation of single trial EPs is to use it as a preprocessing technique prior to averaging. The improvement in SNR might make it possible to compute averages of high quality using less EP trials. This would greatly benefit experimental work since recording of EPs is time and cost consuming.

We compared GMMnoise ($M = 10, N = 1$ just as before), averaging and averaging after application of the GMMnoise method (GMMnoise+ave) on subsets of the "1:3" data set containing 10, 20, 50 and 100 EP trials. Estimates of the noise means μ_b and variances σ_b^2 were based on the respective parameters of the residual noise obtained via averaging of 10, 20, 50 or 100 trials. Results are depicted in Fig. 2(b). Results achieved by averaging are worse for numbers of trials smaller than 50 when compared to results obtained by GMMnoise+ave.

GMMnoise+ave shows considerable improvements of SNR already using only 10 or 20 EP trials.

5 Conclusion

We presented an adaption of the speech analysis method of Gaussian mixture models with integrated noise component (GMMnoise) to the estimation of single trials of evoked potentials (EP). Using a testbed of simulated visually evoked potentials we showed that GMMnoise performs at the level of other state of the art techniques. We also showed that GMMnoise could also be used as a preprocessing method prior to averaging to enable computation of high quality averages using less EP trials.

Up till now we used GMMs with diagonal covariance matrices only. Employing full covariances might allow more accurate modeling of signals. Another possibility for improving GMMnoise would be to account for the non-stationarity of EP signals by switching from atemporal GMMs to Hidden Markov Models (HMM). [Logan 1998] presents an extension of the GMMnoise framework to the non-stationary HMM case where auto-regressive observation densities are used to model signal and noise. Although it was necessary to use artificial data to accurately measure the gain in SNR obtained via GMMnoise the ultimate test would of course be its application to real EP data. An EP data set where considerable trial to trial variance is expected would be an ideal testbed.

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